A new finite element approach for electric field computation at the surface of overhead transmission line conductors

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The purpose of this paper is to introduce a new approach for the computation of electric field at the surface of overhead transmission line conductors through the Finite Element Method. The strategy proposed is based in a spatial transformation, well-known as Kelvin Transformation, resulting in a special way to treat the unbounded domain. This can reduce the domain of studies, allowing an accurate numerical calculation of the electric potential gradients, without the need of geometric simplifications of the conductors' domain. The comparative results with the Charge Simulation Method shows that a more realistic treatment can provide a better understanding of how electric fields operate in the vicinity of real conductors, favoring a more accurate design of transmission lines.

***Index Terms* — Finite Element Analysis, Kelvin Transformation, Open Boundary Quasi Static Electric Field, Surface Voltage Gradient**

I. Introduction

One of the most important consideration in the design of high voltage transmission lines is their corona performance. Since the production of corona effects (like electromagnetic interference, audible noise, gaseous effluents and light) means economic losses, this plays an important role in the electrical planning of transmission lines (TLs), bringing up technical doubts for the adequate choice of cable types, cable gauges and bundle configurations. Due to the constant use of compact structures in urban regions, this situation is true even in lower voltage classes [1].

The electric field at the surface of conductors is the factor which has most influence in corona performance [1]. Therefore, an accurate knowledge of this measure is essential for the TLs design. However, the traditional techniques commonly applied in this calculation, like the Charge Simulation Method (CSM) [2], have become impracticable, since they are based on simplified models, resulting in inaccuracies [3].

Although computational simulations using the Finite Element Method (FEM) can deliver a better support in these cases, like presented in recent studies [3], without the need for too much simplification of existing geometries, some different difficulties are also encountered. The main is the extremely high scale factor, due to the dimension of the towers compared to the low diameter of the cables. Also, as the TLs have an open physical boundary that extends until infinity, fictitious boundaries should be considered, limiting the study domain.

These mutual difficulties will require a careful discretization of the domain in the FEM application. In the cables vicinity, it is necessary to use a very refined element mesh. At the same time, a very extensive element mesh is necessary, considering the interaction with the ground, and also extending to distant regions where the electric field values become negligible. This causes a high generation of elements that must have well-controlled sizes, with a high cost of processing and large use of computing memory, in order to achieve an accurate result for the electric fields at the conductors' surface. Additionally, a special technique must be applied in the fictitious boundary to correctly represent the untreated external domain.

In this way, it is highly desirable to improve the application of FEM in TLs using some technique that allows the domain reduction, minimizing the number of elements in the discretization process. It's also necessary to consider appropriate approaches for the unbounded domain.

Because of these requirements, in order to develop accurate and efficient tools for electric field estimation at the surface of real conductors, this paper aims to introduce a new approach for FEM application in TLs. The proposed technique is based on the Kelvin Transformation [4]. Although other works have applied this technique in the TL domain transformation [5], new modifications are proposed, focusing the analysis only in the conductors' region. This difference is essential for the domain reduction, offering advantages for the FEM application.

II. Proposed Kelvin Transformation

Among several ways of dealing with open boundary problems, the inclusion of fictitious boundaries with the spatial transformation of the outer region is an attractive way recently used in electromagnetic problems. The most commonly used technique is called "Kelvin Transformation" [4], which uses a circular fictitious boundary $'\Gamma_f'$ to enclose the objects of interest in an internal domain $'\Omega_{int}'$. To properly consider the outer region, the infinite remaining domain $'\Omega_{ext}'$ is continuously mapped in another circular closed domain through geometric mapping functions. The technique stipulates that the radius $'a'$ of both circular domains must be equal, so each point (x, y) in the original external domain can be represented by a unique point (ξ, η) in the transformed domain, according to:

$$\xi = (a^2 / r^2) \cdot (x - x_c) + \xi_c \tag{1}$$

$$\eta = (a^2 / r^2) \cdot (y - y_c) + \eta_c, \tag{2}$$

where (x_c, y_c) is the central point in the internal domain (in real coordinates), (ξ_c, η_c) is the central point in the external transformed domain, and $'r'$ is the Euclidian distance between the central point (x_c, y_c) and the external point (x, y):

$$r = \sqrt{(x - x_c)^2 + (y - y_c)^2} \, . \tag{3}$$

With this transformation, moving beyond the fictitious boundary in the real domain, toward the infinity, means moving toward the center of the transformed domain.

To apply the Kelvin Transformation in TLs, this work suggests imposing the virtual boundary close to the conductors, reducing the study domains, as shown in in Fig. 1. So, the ground is entirely assumed in the external transformed domain.

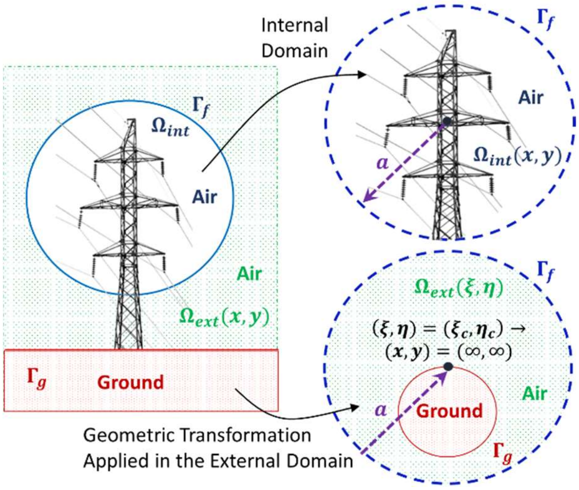

Fig. 1. Kelvin Transformation proposal to FEM applied in TL applications

Once the outer region is transformed, both domains can be normally discretized. Although in each domain the mesh elements will be different, the same nodal points in the circular boundaries $'\Gamma_f'$ must be taken, since the fictitious limit is unique. The electric potentials in these coincident nodes must be equal as well. For the inner domain, the application of FEM does not require additional adjustments. It is identical to that used in the resolution of Poisson equations (with complex electric scalar potential) [6]. However, it is necessary to develop the contribution terms for the elements in external domain. For these elements, the derivative of shape functions $'N_i^e'$ in relation to each coordinate must be modified through the Jacobian matrix terms of the spatial transformation [7]. So, assembling the contribution terms of both domains result in a linear system for the electric potentials in the nodes. Before solving it, it is necessary to apply the Dirichlet boundary condition. Besides the electric potential phasors on the conductors, the reference potential in the center of the transformed external domain $(\phi(\xi_c, \eta_c) = 0)$ must be imposed.

III. Results

The proposed approach is used in the evaluation of a 500 kV TL brazilian existing electrical system. It uses Aluminium Conductor Steel Reinforced (ACSR) cables with 28,74mm diameter in triangular three-conductor bundles. The results of electric field in the surface of conductors is compared between FEM, considering a true cross-section geometry and materials, and CSM, considering an ideal conductor. A homogeneous ground with 100Ωm is also considered in FEM.

In each approach, although the electric field at the conductors' surface seems to follow a same trend, there are errors in the maximum values for all TL conductors, due to the behavior in the straps. The FEM results can prove that, on the surface of a real conductor, the eletric field abruptly varies along each strand surface. The conductor, when correctly modeled with round strands, has a higher maximum surface gradient than that in an ideal conductor model. Although this error is negligible in some conductors (<5%), errors greater than 20% are observed in the central bundle conductors. The comparative behavior in the worst case is shown in Fig. 2. In this case, the maximum superficial electric field value is 12.1 kV/cm for FEM, against 9.2 kV/cm for CSM.

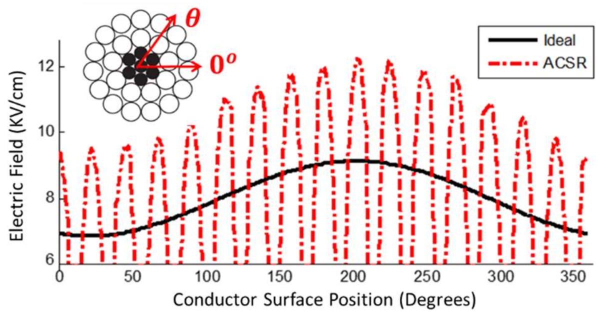

Fig. 2. Electric field at the conductor's surface – Ideal (CSM) x ACSR (FEM)

Similar results can be extended for other TLs using different types of conductors. The extended paper will provide additional results, including comparisons with other classical methods and more details about time consuming in the FEM approach. These results show that FEM can be used to help the design of TLs providing an accurate and computationally efficient analysis of the electric field at the conductors' surface.

Acknowledgment

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